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ON g-BINARY m-OPEN SETS AND MAPS

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ABSTRACT

In this paper we introduce and study the concept g-binary m-open sets and g-binary m-continuity in g-binary topological spaces and investigate various properties.

Keywords: *g-binary open sets, g-binary semi-open sets, g-binary pre-open sets, g-binary m-open sets, g-binary m-continuous functions.*

I. INTRODUCTION

In 2011 A. I. El-Maghrabi and M. A. Al-Johany [1] intreoduced the concept of M-open set in topological spaces and studied the various properties of these sets. Nithyanantha Jothi and P.Thangavelu [16] in 2011 introduced the concept of binary topology between two sets and investigate some of the basic properties. The purpose of this paper is to introduce and study g-binary m-open sets and g-binary m-continuity in g-binary topological spaces and investigate various relationships. Let X and Y are any two non-empty sets. A g-binary topology from X to Y is a binary structure $M_g \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:
$(\emptyset, \emptyset)$ and $(X, X) \in M_g$
If $\{(A_\alpha, B_\alpha); \alpha \in \Delta\}$ is a family of members of M_g, then $(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha) \in M_g$

If M_g is a g-binary topology from X to Y, then the triplet (X, Y, M_g) is called a g-binary topological space and the members of M_g are called the g-binary open subsets of the g-binary topological space (X, Y, M_g). The elements of $X \times Y$ are called the g-binary points (or g-binary sets) of g-binary topological space (X, Y, M_g). Let (X, Y, M_g) be a g-binary topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is g-binary closed in (X, Y, M_g) if $(X\backslash A, Y\backslash B) \in M_g$.

Section 2 deals with the basic concepts of g-binary topological spaces. In section 3 g-binary m-open sets and g-binary m-continuity in g-binary topological spaces are studied and established the relationships. Throughout the paper $\wp(x)$ denotes the power set of x.

II. PRELIMINARIES

Definition 2.1: Let (X, Y, M_g) be a g-binary topological space and $(A, B) \subseteq (X, Y)$. Let ${(A,B)^{1^*}}_g = \bigcap\{A_\alpha: (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and Let ${(A,B)^{2^*}}_g = \bigcap\{B_\alpha: (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $({(A,B)^{1^*}}_g, {(A,B)^{2^*}}_g)$is g-binary closed and $(A, B) \subseteq ({(A,B)^{1^*}}_g, {(A,B)^{2^*}}_g)$. The ordered pair $\left({(A,B)^{1^*}}_g, {(A,B)^{2^*}}_g\right)$ is called g-binary closure of (A, B) and is denoted $gbcl(A, B)$ in the g-binary topology (X, Y, M_g) where $(A, B) \subseteq (X, Y)$.

Proposition 2.1: In a g-binary topological space (X, Y, M_g) if $(A, B) \subseteq (X, Y)$, then $gbcl(A, B)$ is smallest g-binary closed set containing (A, B).

Proposition 2.2: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary closed in (X, Y, M_g) iff $(A, B) = gbcl(A, B)$.

Definition 2.2: Let (X, Y, M_g) be a g-binary topological space and $(A, B) \subseteq (X, Y)$. Let ${(A, B)^{1^0}}_g = \cup \{A_\alpha : (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and Let ${(A, B)^{2^0}}_g = \cup \{B_\alpha : (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.Then $({(A, B)^{1^0}}_g, {(A, B)^{2^0}}_g)$is g-binary open and $({(A, B)^{1^0}}_g, {(A, B)^{2^0}}_g) \subseteq (A, B)$. The ordered pair $({(A, B)^{1^0}}_g, {(A, B)^{2^0}}_g)$ is called g-binary interior of (A, B) and is denoted by $gbint(A, b)$.

Proposition 2.3: In a g-binary topological space (X, Y, M_g) if $(A, B) \subseteq (X, Y)$, then $gbint(A, B)$ is largest g-binary open set contained in (A, B).

Proposition 2.4: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary open in (X, Y, M_g) iff $(A, B) = gbint(A, B)$.

Definition 2.3: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called
g-binary semi-open if $(A, B) \subseteq gbcl(gbint(A, B))$.
g-binary pre-open if $(A, B) \subseteq gbint(gbcl(A, B))$.
g-binary α-open if $(A, B) \subseteq gbint\left(gbcl\left(gbint(A, B)\right)\right)$
g-binary β-open if $(A, B) \subseteq gbcl(gbint(gbcl(A, B)))$

Definition 3.4: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be
g-binary continuous if $f^{-1}(A, B)$ is g-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary semi-continuous if $f^{-1}(A, B)$ is g-semi-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary pre-continuous if $f^{-1}(A, B)$ is g-pre-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary α-continuous if $f^{-1}(A, B)$ is g-α-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary β-continuous if $f^{-1}(A, B)$ is g-β-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

III. G-BINARY M-OPEN SETS AND MAPS

In this section we will introduce m-open sets and m-continuity in g-binary topological spaces. Let (X, Y, M_g) be a g-binary topological space. The g-binary point is said to be in the δ-closure (resp. θ-closure) of a subset $(A, B) \subseteq (X, Y)$ if for each g-binary open neighborhood (U, V) of that g-binary point we have $gbint\left(gbcl(U, V)\right) \cap (A, B) \neq \emptyset$ and (resp. $gbcl(U, V) \cap (A, B) \neq \emptyset$). The δ-closure (resp. θ-closure) of a subset $(A, B) \subseteq (X, Y)$ is denoted by $gbcl_\delta(A, B)$ resp. $gbcl_\theta(A, B)$. A subset $(A, B) \subseteq (X, Y)$ is called δ-closed (resp. θ-closed) if $(A, B) = gbcl_\delta(A, B)$ (resp. $(A, B) = gbcl_\theta(A, B)$. The complement of δ-closed (resp. θ-closed) set is called δ-open (resp. θ-open). The families of all δ-open (resp. θ-open) subsets (X, Y, M_g) forms g-binary topology denoted by δM_g (resp.θM_g). From the definitions it follows easily that $\theta M_g \subseteq \delta M_g \subseteq M_g$.

Definition 3.1: Let (X, Y, M_g) be an g-binary topological space and (A, B) be a subset of $\wp(X) \times \wp(Y)$, then
$gbcl_\delta(A, B) = \{(x, y) \in \wp(X) \times \wp(Y): gbint\left(gbcl(U, V)\right) \cap (A, B) \neq \emptyset, (U, V) \in M_g \text{ and } (x, y) \in (U, V)\}$
$gbcl_\theta(A, B) = \{(x, y) \in \wp(X) \times \wp(Y): gbcl(U, V) \cap (A, B) \neq \emptyset, (U, V) \in M_g \text{ and } (x, y) \in (U, V)\}$

Definition 3.2: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called
g-binary δ-pre-open set if $(A, B) \subseteq gbint(gbcl_\delta(A, B))$
g-binary θ-semi-open set $(A, B) \subseteq gbcl(gbint_\theta(A, B))$

Definition 3.3: Let (X, Y, M_g) be an g-binary topological space and (A, B) be a subset of $\wp(X) \times \wp(Y)$, then (A, B) is called
g-binary m-open set if $(A, B) \subseteq gbcl\left(gbint_\theta(A, B)\right) \cup gbint(gbcl_\delta(A, B))$
g-binary m-open set if $(A, B) \supseteq gbint\left(gbcl_\theta(A, B)\right) \cap gbcl(gbint_\delta(A, B))$

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Proposition 3.1: In a g-binary topological space (X, Y, M_g)
Every g-binary θ-semi-open set is g-binary m-open.
Every g-binary δ-pre-open set is g-binary m-open.

Proof: Obvious
Remark 3.1: Converse of Proposition 3.1 is not true in general as shown in Example 3.1

Example 3.1: Let $X = \{1,2,3\}$ and $Y = \{a, b, c\}$. Then $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a, b\}), (\{2,3\}, \{c\}), (\{1,3\}, \{Y\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Therefore the set $(\{1,3\}, \{a, b\})$ is g-binary m-open but not g-binary θ-semi-open or g-binary δ-pre-open.

Definition 3.4: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be g-binary m-continuous if $f^{-1}(A, B)$ is g-m-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Example 3.2: Let $Z = \{1, 2, 3,4\}, X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3\}$.Then $\tau = \{\emptyset, \{1\}, \{3,4\}, \{1,2,4\}, \{1,3,4\} Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1) = f(3)$ and $f(2) = f(4) = (a_2, \emptyset)$. Now $f^{-1}(\emptyset,$ ∅=∅, f−1(a1, b1)={1,3}, f−1(a2, b2)={∅}, f−1(a2, Y)={∅} and f−1X, Y=Z. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-m-open in (Z, τ). Hence f is g-binary m-continuous function.

Definition 3.4: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be g-binary θ-semi-continuous if $f^{-1}(A, B)$ is g-θ-semi-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Example 3.3: Let $Z = \{1, 2, 3,4\}, X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3\}$.Then $\tau = \{\emptyset, \{1\}, \{3,4\}, \{1,2,4\}, \{1,3,4\} Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_2, \emptyset) = f(3)$ and $f(2) = f(4) = (a_1, b_1)$. Now $f^{-1}(\emptyset,$ ∅=∅, f−1(a1, b1)={2,4}, f−1(a2, b2)={∅}, f−1(a2, Y)={∅} and f−1X, Y=Z. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-θ-semi-open in (Z, τ). Hence f is g-binary θ-semi-continuous.

Proposition 3.2: Every g-binary θ-semi-continuous function in g-binary topological space is g-binary m-continuous.
Proof: Obvious from the definition

Remark 3.2: Converse of Proposition 3.2 is not true in general as shown in Example 3.4

Example 3.4: In Example 3.2 f is g-binary m-continuous function but not g-binary θ-semi-continuous because the set {1,3} is g-m-open in (Z, τ) but not g-θ-semi-open.

Definition 3.5: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be g-binary δ-pre-continuous if $f^{-1}(A, B)$ is g-δ-pre-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Example 3.5: Let $Z = \{1, 2, 3,4\}, X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3\}$.Then $\tau = \{\emptyset, \{1\}, \{3,4\}, \{1,2,4\}, \{1,3,4\} Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_2, \emptyset) = f(2)$ and $f(3) = f(4) = (a_1, b_1)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{3,4\}$, $f^{-1}(\{a_2\}, \{b_2\}) = \{\emptyset\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{\emptyset\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-δ-pre-open in (Z, τ). Hence f is g-binary δ-pre-continuous.

Proposition 3.3: Every g-binary δ-pre-continuous function in g-binary topological space is g-binary m-continuous.

Proof: Obvious from the definition
Remark 3.3: Converse of Proposition 3.3 is not true in general as shown in Example 3.6

Example 3.6: In Example 3.2 f is g-binary m-continuous function but not g-binary δ-pre-continuous because the set $\{1,3\}$ is g-m-open in (Z, τ) but not g-δ-pre-open.

Remark 3.4: Every g-binary continuous function in g-binary topological space is g-binary m-continuous but not converse as shown in Example 3.7.

Example 3.7: In Example 3.2 f is g-binary m-continuous function but not g-binary continuous because the set $\{1,3\}$ is g-m-open in (Z, τ) but not g-open.

Remark 3.5: Every g-binary pre-continuous function in g-binary topological space is g-binary m-continuous but not converse as shown in Example 3.8.

Example 3.8: Let $Z = \{1, 2, 3, 4\}$, $X = \{a_1, a_2, a_3\}$ and $Y = \{b_1, b_2, b_3\}$.Then $\tau = \{\emptyset, \{4\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{2,3,4\}, \{1,2,4\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \rightarrow X \times Y$ by $f(1) = (a_1, b_1) = f(4)$ and $f(2) = f(3) = (a_2, \emptyset)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1,4\}$, $f^{-1}(\{a_2\}, \{b_2\}) = \{\emptyset\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{\emptyset\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-m-open in (Z, τ). Hence f is g-binary m-continuous but not g-binary pre-continuous because the set $\{1,4\}$ is g-m-open but not g-pre-open in (Z, τ)

Remark 3.6: Every g-binary semi-continuous function in g-binary topological space is g-binary m-continuous but not converse as shown in Example 3.9.

Example 3.9: In Example 3.8 f is g-binary m-continuous but not g-binary semi-continuous because the set $\{1,4\}$ is g-m-open but not g-semi-open in (Z, τ).

Remark 3.7: Every g-binary β-continuous function in g-binary topological space is g-binary m-continuous but not converse as shown in Example 3.10.

Example 3.10: In Example 3.8 f is g-binary m-continuous but not g-binary β-continuous because the set $\{1,4\}$ is g-m-open but not g-β-open in (Z, τ).

Remark 3.8: Every g-binary α-continuous function in g-binary topological space is g-binary m-continuous but not converse as shown in Example 3.11.

Example 3.11: In Example 3.8 f is g-binary m-continuous but not g-binary α-continuous because the set $\{1,4\}$ is g-m-open but not g-α-open in (Z, τ).
From the above discussion we have the following result:
g-binary θ-semi-continuous $\Longrightarrow (\not\Leftarrow)$ g-binary m-continuous
g-binary δ-pre-continuous $\Longrightarrow (\not\Leftarrow)$ g-binary m-continuous
g-binary continuous $\Longrightarrow (\not\Leftarrow)$ g-binary m-continuous
g-binary semi-continuous $\Longrightarrow (\not\Leftarrow)$ g-binary m-continuous
g-binary pre-continuous $\Longrightarrow (\not\Leftarrow)$ g-binary m-continuous

g-binary α-continuous ⟹ (⇍) g-binary m-continuous
g-binary β-continuous ⟹ (⇍) g-binary m-continuous

IV. CONCLUSION

g-binary m-open sets and g-binary m-continuity in g-binary topological spaces is introduced and studied. Further different relationships between these functions are investigated

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